

# Supporting Information for “Tipping elements and climate-economic shocks: Pathways toward integrated assessment”

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### Text S1. Supporting methods for Figure 2

The systems shown in Figure 2 are driven by the following equations:

$$y_{eq}(x) = A \left( 1 - 1 / \left[ 1 + \exp \left( - \frac{x - x_0}{\sigma} \right) \right] \right) - \beta x + c \quad (1)$$

$$\dot{x}(y) = F(t) - \gamma(y - y_0) \quad (2)$$

$$\frac{dy}{dt} = \frac{y_{eq} - y}{\tau} \quad (3)$$

where  $F(t)$  represents the exogenous forcing as a function of time (Figure 2a),  $y_{eq}$  represents the equilibrium response as a function of a state variable  $x$  related to forcing (Figure 2b), and  $y$  represents the realized response of the system.  $A$  scales the non-linear system response,  $\sigma$  scales the width of the sigmoidal equilibrium response,  $\beta$  is the linear response

of the system to the state variable  $x$ ,  $\tau$  is the timescale over which the system approaches equilibrium, and  $\gamma$  represents the feedback of  $y$  onto  $x$ . In Figure 2c-d,  $\gamma = 0$ . In Figure 2e-f,  $\tau \rightarrow 0$  so  $y = y_{eq}$ .

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